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EXPERIMENTAL STUDY OF SUPERSONIC THREE-DIMENSIONAL JETS

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UDC 533.17

The interest in the study of three-dimensional jets, i.e., jets in which the three-dimensional character of flow is due to the form of the outlet section of the nozzle [1], stems from their increasing practical value. For example, such nozzles are used in modern supersonic aircraft [2], in the gas-processing industry [3], and in other applications.

There have been relatively few experimental studies of the propagation of three-dimensional jets; of the studies that have been conducted, we can note [4-6], with the latter being the most complete.

Here, we experimentally study the shock-wave structure and parameter distribution in supersonic underexpanded jets of cold air (T_0 ~ 290 K) discharged into the atmosphere (p_∞ ~ 0.1 MPa) from rectangular sonic nozzles. We used schlieren visualization of the flow and we measured the total head on the jet axis. Empirical relations were obtained to determine the position of the central discontinuity in three-dimensional jets and the Mach-number distribution on the axis. The results are compared with the data in [6].

In our experiments, we used sonic nozzles with a rectangular edge and a ratio of sides of the rectangle λ equal to 1, 2, 3, 5, and 10. This ratio is referred to below as the elongation of the nozzle edge. The size of the lesser side was 6-12 mm. The nozzle took the form of a rectangular opening in the end of a cylinder with an inside diameter of 80 mm. The nozzle had shaped subsonic and equalizing plane-parallel sections about 4 mm long. The

Leningrad. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 75-80, May-June, 1988. Original article submitted January 30, 1987.

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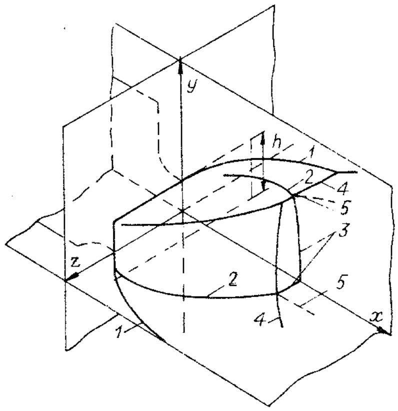

Fig. 1

stagnation pressure p_0 was recorded by standard manometers, atmospheric pressure was measured with a barometer, and the degree of underexpansion n was determined from the formula $n = p_0/p_\infty((\gamma + 1)/2)^{-\gamma/(\gamma-1)}$ to within 3%. Here, $\gamma = 1.4$. Visualization of the shock-wave structure was accomplished with an IAB-451 optical device in two mutually perpendicular planes parallel to the sides of the nozzle edge. The visualization was recorded on photographic film and the linear dimensions were determined on an MMI-2 tool microscope with an error generally no greater than 3%. The total heads were measured with a cylindrical probe having an outside diameter of 2 mm. The overhang of the probe from the holder was 25 mm. The holder had the form of a wedge with an angle of 30° at the vertex. As a result of vibration of the manometer pointer during the measurements, the actual error of measurement of total head was 3-5%.

Figure 1 shows a diagram of the flow in a three-dimensional jet, with the main notation and terms having been borrowed from [4, 5]. Here, 1 is the boundary of the jet, 2-4 are suspended, central, and reflected shocks, and 5 is a tangential discontinuity.

Figure 2 shows the evolution of the shock-wave structure with an increase in n (the notation is the same as in Fig. 1). As noted in [5], formation of the wave structure in the principal plane begins with the formation of a central shock which is convex in the nozzle direction (Fig. 2a). An increase in n is accompanied by replacement of this form of shock wave by an X-shaped shock (b) and then by irregular interaction of suspended shocks (c). Here, the curvature of the central shock acquires increasingly greater convexity in the direction of the nozzle (d) and its size increases monotonically. In the longitudinal plane, the suspended shock is formed almost at the nozzle edge for all values of n. The size of the central shock in this plane at small λ ($\lesssim 4$) also increases monotonically with n, while at large λ it initially decreases and then increases. The different character of change in the size of the central shock in the principal and longitudinal planes with a change in n leads to reorientation of the maximum transverse dimension of the wave structure with an increase in n. A similar result was obtained in the theoretical study [1] relative to the maximum size of the boundary of a jet leaving a nozzle with an elliptical edge.

The range (with respect to λ and n) of our tests and analysis of the results show that, from a qualitative viewpoint, the configurations shown in Fig. 2 exhaust the range of three-dimensional wave structures seen in supersonic jets flowing from rectangular nozzles.

The most commonly used scale parameter of shock-wave structure is the distance from the nozzle edge to the central shock. Experimental values of this distance $x_s^0 = x_s/h$ are shown by the clear symbols in Fig. 3, while the dark symbols show results from [6]. Solid lines 1 and 2 show empirical relations for the distances to the Mach disk x_M^0 in axisymmetric jets [7]

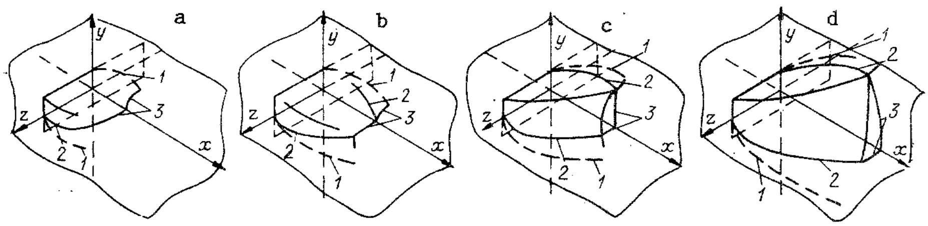

Fig. 2

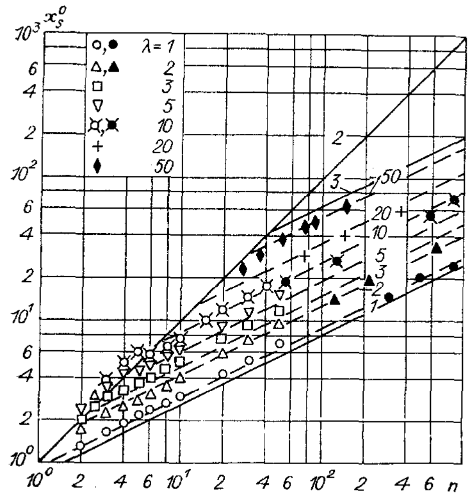

Fig. 3

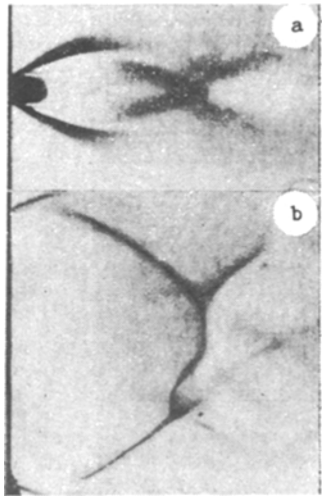

Fig. 4

($x_M^0 = 0.69 M_a \sqrt{\gamma n}$) and the Riemann wave x_R^0 in plane jets [8] ($x_R^0 = M_a n$) with the corresponding experimental values of the parameters ($M_a = 1$, $\gamma = 1.4$).

It follows from Fig. 3 that with small values of n, for each λ the distance to the central shock is independent of λ and is equal in a first approximation to the distance to the Riemann wave. At large n (greater than a certain value n_* characteristic for each λ), the experimental values of x_s^0 are stratified with respect to the parameter λ. To construct the empirical relation for x_s^0 in this region of n ($n > n_*$), we used the results obtained in [9] from an analysis of the similarity parameters of highly underexpanded jets ($n \gg 1$). It follows from the comparison that at $n \gg 1$, similarity exists between flows with the same integral gas characteristics at the nozzle edge but with different field structures. Thus, it can be expected that if we take the characteristic dimension of the edge of a nozzle of arbitrary form to be the diameter of a circle of the same area $d = \sqrt{4\lambda/\pi}h$, then with an increase in n the parameter $x_s^0 = x_s/d$ will asymptotically approach the values corresponding to the axisymmetric case. Here, the relation for x_s has the form $x_s^0 = x_s/h = (4/\pi)^{0.5}\lambda^{0.5}x_M^0$ ($x_M^0 = x_M/d$). At small λ (≤5), it accurately describes the distribution of empirical values of x_s^0 not only at $n \gg 1$, but throughout the range $n > n_*$ if we use the Lewis–Carlson relation [7]. At large λ, the agreement with experimental results is poorer (for example, curve 3 for λ = 50 in Fig. 3).

To approximate the experimental data in the range λ = 1-50, we propose the relation

$$x_s^0 = (4/\pi)^{0.5}\lambda^{0.5-\varepsilon(\lambda)}x_M^0, \quad \varepsilon(\lambda) = 0.03 \lg \lambda, \tag{1}$$

shown by the dashed lines in Fig. 3.

The value of n_* at which the character of the dependence of x_s^0 on n changes is obviously determined from the equality $x_R^0(n_*) = (4/\pi)^{0.5}\lambda^{0.5-\varepsilon(\lambda)}x_M^0(n_*)$. At $\gamma = 1.4$ and λ = 2-50, this equality is approximated to within 6% by the following expression

$$n_*(\lambda) = 0.63\lambda + 0.60. \tag{2}$$

Thus, at $n \le n_*$, the distance from the nozzle edge to the central shock on the axis of a three-dimensional jet is determined by the formula $x_s^0 = M_a n$; with $n > n_*$ – by Eq. (1).

For the nozzles studied here (λ ≤ 10), with $n \simeq n_*$, regular reflection of the suspended shock is replaced by irregular reflection. In fact, the relation $x_s^0(n)$ in the neighborhood of n_* is somewhat more complex in character. For example, at λ = 10, it is nonmonotonic [$x_s^0(5) < x_s^0(4)$]. Figure 4 shows schlieren photographs of the shock-wave structure in the principal (a) and longitudinal (b) planes at the moment of change in the character of reflection of the suspended shock [regular (Fig. 2b) into irregular (Fig. 2d)]. This figure also illustrates the beginning of the nonmonotonic change in the parameter x_s^0 in relation to n.

The flow region in three-dimensional jets that is bounded by the suspended shock and the central shock is obviously a region of isentropic expansion, and the universal form of representation of the parameters in this region is the Mach-number distribution. Primary data on the Mach-number distribution on the axis of jets for different nozzle elongations is shown

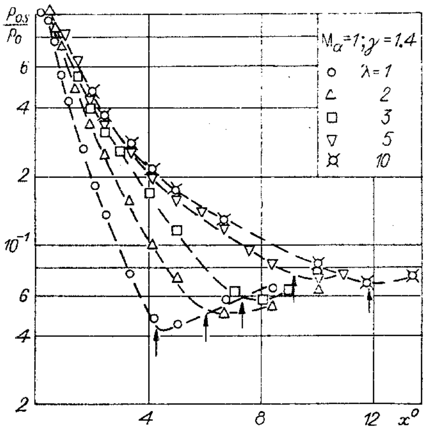

Fig. 5

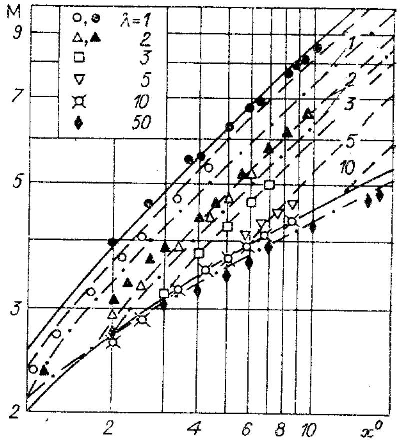

Fig. 6

in Fig. 5 in the form of normalized distributions of the measured total head p_{0s}/p_0 on the jet axis with n = 20. The arrows indicate the position of the central shock with this value of n according to the results of visualization of the shock-wave structure. It is apparent that the position of the shock nearly coincides with the minimum on the total-pressure curve. However, insufficient experimental data in the vicinity of the shock prevent us from making a quantitative estimate of this correlation.

The Mach-number distributions corresponding to the experimental values of total head are shown in Fig. 6 by the clear symbols. The dark symbols show results from [6], while the solid lines show axial distributions of the Mach number in axisymmetric (top line – results from [10]) and plane (bottom line) jets. It is evident (as was noted in [6]) that the distribution $M(\lambda, x^0)$ in the neighborhood of the nozzle is independent of λ and coincides with the distribution in the plane jet. It then acquires the character of axisymmetric flow, i.e., the relation $M(\lambda, x^0)$ is qualitatively similar to the relation $x_s^0(\lambda, x^0)$ examined earlier. The author of [6] proposed empirical relations for $M(\lambda, x^0)$ which take into account the above-noted character of the distribution of experimental results. These relations have the form

$$\frac{T_0}{T} = k\,(x^0)^{\gamma-1} \quad \text{for} \quad x^0 \leqslant x^0_*; \tag{3}$$

$$\frac{T_0}{T} = kx^0\lambda^{-(\gamma-1)}\,(x^0)^{2(\gamma-1)} \quad \text{for} \quad x^0_* < x < x^0_s, \tag{4}$$

where k = 1.89; $x^0_* = \lambda$; T and T_0 are experimental values of the static temperature of the flow and the stagnation temperature, while the Mach number is connected with T_0/T by the isentropic relation $M^2 = 2(\gamma - 1)^{-1}(T_0/T - 1)$. Figure 6 shows these relations with dot–dash curves.

One consequence of the limiting simplicity of approximate expressions (3) and (4) is their relatively low accuracy. For example, for $\lambda = 1$, the error of M is 15-17%.

We propose an approximate formula with an error no greater than 3% for M in the range $1 \leqslant x^0 \leqslant 20, 1 \leqslant \lambda \leqslant 10, \gamma = 1.4, M_a = 1$:

$$\frac{T_0}{T} = k_1\,(x^0)\,(x^0)^{0.4} \quad \text{for} \quad x^0 \leqslant x^0_*; \tag{5}$$

$$\frac{T_0}{T} = k_2\,(x^0)\left(\frac{\pi}{4}\right)^{0.4}\lambda^{0.5}\,(x^0)^{0.8} \quad \text{for} \quad x^0_* < x^0 < x^0_s. \tag{6}$$

Here, $k_1(x^0) = 2.1 - \frac{0.6}{x^0+1}$; $k_2(x^0) = 2.66 - \frac{2}{x^0+5}$; $x^0_* = 1.4(\lambda - 0.7)$.

The coefficients k_1 and k_2 were chosen so that Eqs. (5) and (6) closely approximate numerical results for the plane and axisymmetric cases [10] shown by solid lines in Fig. 6. For example, approximations (6), having the form $T_0/T = k_2(x^0)(x^0)^{0.8}$ in the axisymmetric case, have an error no greater than 1% for all $x^0 \geq 1$. The well-known approximation from [11] has the same error for $x^0 \lesssim 3.5$. Empirical formulas (3) and (4) do not provide such agreement with numerical results. Equations (6) are shown by dashed lines in Fig. 6.

For nozzles with edges characterized by a low degree of elongation ($\lambda - 1 \ll 1$), it is natural to expect that the distribution of the parameters on the axis (except perhaps for the region closest to the edge) will coincide with the distribution in an axisymmetric jet if we use the diameter of an equivalent circular nozzle d as the linear scale. The experimental results obtained for a square nozzle support this proposition. At $\lambda = 1$, approximation (6) coincides with the axisymmetric distribution if we put $x^0 = x/d$, and its agreement with the experimental data throughout the investigated range of x^0 is very good beginning with $x^0 = 1$ (the error here is no greater than 2%, i.e., is commensurate with the experimental error). At $\lambda = 10$, the agreement between the experimental data and the results of numerical calculations in the neighborhood of the nozzle edge $x^0 < x^0_*$ is also quite good. The agreement of the experimental data and numerical results in two limiting cases — axisymmetric and plane — suggests that the difference — which reaches 8% — between our experimental results and the data in [6] can be attributed to the errors of the latter.

Thus, we have derived empirical relations which determine the position of the central shock or the point of regular interaction of suspended shocks in supersonic underexpanded jets leaving a sonic nozzle with a rectangular edge. We also found a relation to determine the Mach numbers on the axis of such jets in the region of free expansion.

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